EXERCISE – V

JEE PROBLEMS

1.(a) A solution of the differential equation,

$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is **[JEE 99,2 + 3 + 10]**

(A) $y = 2$ (B) $y=2x$ (C) $y=2x-4$ (D) $y=2x^2-4$

(b) The differential equation representing the family of curves, $y^2 = 2c\left(x+\sqrt{c}\right)$, where c is a positive parameter, is of

(A) order 1 (B) order 2 (C) degree 3 (D) degree 4

(c) A curve passing through the point (1, 1) has the property that the perpendicular distance of the origin from the normal at any point P of the curve is equal to the distance of P from the x-axis. Determine the equation of the curve.

2. Solve the differential equation,

$(x^2 + 4y^2 + 4xy)\, dy = (2x + 4y + 1)dx$. **[REE 99,6]**

3. A country has a food deficit of 10%. Its population grows continuously at a rate of 3%. Its annual food production every year is 4% more than that of the last year. Assuming that the average food requirement per person remains constant, prove that the country will become self-sufficient in food after 'n' years, where 'n' is the smallest integer bigger than or equal to, $\frac{\ell n\, 10 - \ell n\, 9}{\ell n\,(1.04) - 0.03}$. **[JEE 2000 (Mains),10]**

4. A hemispherical tank of radius 2 metres is initially full of water and has an outlet of 12 cm^2 cross sectional area at the bottom. The outlet is opened at some instant. The flow through the outlet is according to the law $V(t) = 0.6\sqrt{2gh(t)}$, where V(t) and h(t) are respectively the velocity of the flow through the outlet and the height of water level above the outlet at time t, and g is the acceleration due to gravity. Find the time it takes to empty the tank. **[JEE 2001 (Mains), 10]**

5. Find the equation of the curve which passes through the origin and the tangent to which at every point (x, y) has slope equal to $\frac{x^4 + 2xy - 1}{1 + x^2}$.

[REE 2001 (Mains), 3]

6. Let $f(x)$, $x \ge 0$, be a nonnegative continuous function, and let $F(x) = \int_0^x f(t)dt$, $x \ge 0$. If for some $c > 0$, $f(x) \le cF(x)$ for all $x \ge 0$, then show that $f(x) = 0$ for all $x \ge 0$. **[JEE 2001 (Mains), 5]**

7.(a) A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant = k > 0). Find the time after which the cone is empty. **[JEE 2003 (Mains), 4 + 4]**

(b) If $P(1) = 0$ and $\frac{dP(x)}{dx} > P(x)$ for all $x \ge 1$ then prove that $P(x) > 0$ for all $x > 1$.

8.(a) If $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$, $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ equals **[JEE 2004 (Scr.)]**

(A) 1 (B) 1/2 (C) 1/3 (D) 1/4

(b) A curve passes through (2, 0) and the slope of tangent at point P (x, y) equals $\frac{(x+1)^2 + y - 3}{(x+1)}$. Find the equation of the curve and area enclosed by the curve and the x-axis in the fourth quadrant.

[JEE 2004 (Mains)]

9.(a) The solution of primitive integral equation $(x^2 + y^2)dy = xy\, dx$, is $y = y(x)$. If $y(1) = 1$ and $y(x_0) = e$, then x_0 is

(A) $\sqrt{2(e^2-1)}$ (B) $\sqrt{2(e^2+1)}$

(C) $\sqrt{3}\, e$ (D) $\sqrt{\frac{e^2+1}{2}}$

(b) For the primitive integral equation $ydx + y^2dy = xdy$; $x \in R$, $y > 0$, $y = y(x)$, $y(1) = 1$, then $y(-3)$ is **[JEE 2005 (Scr.)]**

(A) 3 (B) 2 (C) 1 (D) 5

(c) If length of tangent at any point on the curve $y = f(x)$ intercepted between the point and the x-axis is of length 1. Find the equation of the curve. **[JEE 2005 (Mains)]**

10. A tangent drawn to the curve, $y = f(x)$ at $P(x, y)$ cuts the x-axis and y-axis at A and B respectively such that BP : AP = 3 : 1, given that $f(1) = 1$, then **[JEE 2006, 5]**

(A) equation of the curve is $x\frac{dy}{dx} - 3y = 0$

(B) equation of curve is $x\frac{dy}{dx} + 3y = 0$

(C) curve passes through (2, 1/8)

(D) normal at (1, 1) is $x + 3y = 4$

11.(a) Let f(x) be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim\limits_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then f(x) is **[JEE 2007, 3 + 3]**

(A) $\frac{1}{3x} + \frac{2x^2}{3}$ (B) $\frac{-1}{3x} + \frac{4x^2}{3}$ (C) $\frac{-1}{x} + \frac{2}{x^2}$ (D) $\frac{1}{x}$

(b) The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

(A) variable radii and a fixed centre at (0, 1)

(B) variable radii and a fixed centre at (0, –1)

(C) fixed radius 1 and variable centres along the x-axis.

(D) fixed radius 1 and variable centres along the y-axis.

12. Let a solution $y = y(x)$ of the differential equation, $x\sqrt{x^2-1}\,dy = y\sqrt{y^2-1}\,dx = 0$ satisfy $y(2) = \frac{2}{\sqrt{3}}$.

STATEMENT-1 : $y(x) = \sec\left(\sec^{-1}x - \frac{\pi}{6}\right)$ **[JEE 2008, 3]**

and

STATEMENT-2 : y (x) is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1 - \frac{1}{x^2}}$

(A) Statement-1 is true, Statement-2 is true ; Statement-2 is correct explanation for Statement-1.

(B) Statement-1 is true, Statement-2 is true ; Statement-2 is **NOT** a correct explanation for Statement-1.

(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is false, Statement-2 is true.

13. If. y (x) satisfies the differential equation $y' - y \tan x = 2x \text{ sex } x$ and $y(0) = 0$, then **[JEE 2012]**

(A) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$ (B) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$

(C) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$ (D) $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

Sol.